LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **NOVEMBER 2013**

MT 4502/MT 4500 – MODERN ALGEBRA

Date : 13/11/2013 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

PART A

ANSWER ALL THE QUESTIONS $(\mathbf{10} \times \mathbf{2} = \mathbf{20}\ \boldsymbol{marks})$

1. Define Partially ordered sets.
2. If H and K are subgroup of G, then show that H $\cap$ K is also a subgroup of G.
3. Find all the generators of the group Z_7.
4. Show that every subgroup of an abelian group is normal.
5. Define isomorphic groups with an example.
6. Determine which of the following is /are even and odd permutation

   i) $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}$ ii) $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 4 & 2 \end{pmatrix}$

7. Define a ring with an example.
8. If F is a field, then prove that its only ideals are (0) and F itself.
9. Find the characteristic of an integral domain Z_5.
10. Find all the units in $z[i] = \{x + iy / x, y \in Z\}$.

PART-B

ANSWER ANY FIVE QUESTIONS: $(\mathbf{5} \times \mathbf{8} = \mathbf{40}\ \boldsymbol{marks})$

11. Show that a non empty subset H of a group G is a subgroup of G if and only if $a, b \in H$ implies that $ab^{-1} \in H$.
12. State and prove Lagrange's theorem.
13. If G is a group and N is a normal subgroup of G, then prove that G / N is also a group under the product of subsets of G.
14. State and prove first isomorphism theorem.
15. State and prove Cayley's theorem.
16. Show that every finite integral domain is a field.
17. Let R be a commutative ring with unity, and P an ideal of R, then prove that P is a prime ideal of R if and only if R / P is an integral domain.
18. Let R be a Euclidean ring. Show that any two elements a and b in R have a greatest common divisor d which can be expressed in the form $d = \lambda a + \mu b$ for λ, μ in R.

PART- C

ANSWER ANY TWO QUESTIONS $(\mathbf{2 \times 20 = 40}\ \boldsymbol{marks})$

19. a) If H and K are subgroups of G,then prove that HK is a subgroup of G if and only if HK=KH.

    b) Show that a subgroup N of a group G is a normal subgroup of G if and only if $gNg^{-1} = N$ for all $g \in G$.

20. a) Show that every subgroup of cyclic group is cyclic.

    b.)If H is the only subgroup of order O(H) in the finite group G, prove that H is a normal subgroup of G.

21. a) Prove that any infinite cyclic group G is isomorphic to the group Z of integers under addition.

    b) If G is a group, then prove that A(G) the set of automorphisms of G is also a group.

22. a) State and prove the fundamental homomorphism of rings.

    b) Show that an ideal of the Euclidean ring R is a maximal ideal if and if it is generated by a prime element of R .